Daniel Boer*
Dept. of Physics and Astronomy, Vrije Universiteit Amsterdam,
De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

Rainer Jakob†
Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany

Marco Radici‡
Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, and
Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy
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We study the process of electron-positron annihilation into back-to-back jets, where in each jet a pair of hadrons is detected. The orientation of these two pairs with respect to each other can be used to extract the interference fragmentation functions in a clean way, for instance from BELLE or BABAR experiment data. This is of relevance for studies of the transversity distribution function.

In particular, we focus on two azimuthal asymmetries. The first one has already been studied by Artru and Collins, but is now expressed in terms of interference fragmentation functions. The second asymmetry is new and involves a function that is related to longitudinal jet handedness. This asymmetry offers a different way of studying handedness correlations.

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I. INTRODUCTION

Interference fragmentation functions (IFFs) have been suggested as a means to access transversity [1] via single spin asymmetries in $e\,p^\uparrow$ and $p\,p^\uparrow$ processes, in which the proton is transversely polarized. Transversity is a measure of how much of the transverse polarization of a proton is transferred to its quarks. It is a helicity-flip (or chiral-odd) distribution function that is very hard to measure and thus far no extraction from data is available. To become sensitive to the transverse spin of quarks inside a transversely polarized proton one can follow two main routes. The first one is to use another transversely polarized hadron (in initial or final state), like in the Drell-Yan process $p^\uparrow\,p^\uparrow \to \ell\,\bar{\ell}\,X$ or in polarized Λ hyperon production. The second route is to exploit the possibility that the direction of the transverse polarization of a fragmenting quark may somehow be encoded in the distribution of hadrons inside the resulting jet. For instance, the Collins effect [2] describes the case where the distribution of a hadron inside the jet follows a $\boldsymbol{k}_T \times \boldsymbol{s}_T$ behaviour, where $\boldsymbol{k}_T$ is the transverse momentum of the quark compared to the hadron and $\boldsymbol{s}_T$ is the transverse polarization of the fragmenting quark. Here, transverse means orthogonal to the quark (or, equivalently, jet) direction. Due to the transverse momentum dependence, the Collins effect is a very challenging observable both theoretically and experimentally, and an alternative is formed by the IFFs which describe the distribution of two hadrons inside the jet. The idea is that the orientation of two hadrons with respect to each other and to the jet direction, is an indicator for the transverse spin direction of the quark. Such a correlation is expected to occur due to the strong final state interactions between the two hadrons: different partial waves can interfere and this is expected to give rise to nonvanishing, nontrivial fragmentation functions, the two-hadron IFFs.

As with all proposals to measure transversity, a second unknown quantity is introduced, which needs to be measured separately. For the nontrivial fragmentation functions, such as the Collins function and the two-hadron IFFs, the cleanest extraction is from two-jet events in the electron-positron annihilation process. Here, we will present the leading twist, fully differential cross section for the process $e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)X$ in terms of products of two-hadron fragmentation functions.

In Ref. [3], the leading twist, transverse momentum dependent two-hadron IFFs have been listed. There are two chiral-odd and one chiral-even IFF, but upon integration over the *quark* transverse momentum dependence only one

*Electronic address: dboer@nat.vu.nl
†Electronic address: rainer.jakob@physik.uni-wuppertal.de
‡Electronic address: marco.radici@pv.infn.it

chiral-odd IFF survives (called $H_1^{\sphericalangle}$), discussed at several places in the literature [4, 5, 6, 7]. The relation among the various approaches and to the ρ fragmentation functions (for the two hadrons being two pions) is extensively discussed in Ref. [8], where the two-hadron final system is expanded in relative partial waves and a new contribution involving the transversity at leading twist is identified.

Here, we will discuss the consequences of all three leading twist IFFs occurring in the process $e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X$; we find that upon integrating the differential cross section over the *observed* transverse momentum one is actually not only left with the transverse momentum integrated chiral-odd IFF $H_1^{\sphericalangle}$, but there is also an asymmetry that is governed by the chiral-even IFF, integrated, but weighted, over the transverse momentum:

$$G_1^{\perp}(z, M_h^2) \equiv \int d\xi \int d\phi_R \int d\boldsymbol{k}_T\, \boldsymbol{k}_T \cdot \boldsymbol{R}_T\; G_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T), \tag{1}$$

where $\boldsymbol{R}_T$ is the transverse part of the relative momentum between the two hadrons and $\boldsymbol{k}_T$ is the quark transverse momentum (see Sec. II for the explicit definitions of the above quantities). This function is related (but not identical) to longitudinal jet handedness and its resulting asymmetry will be discussed in detail below (see Sec. V).

The asymmetry involving the transverse momentum integrated chiral-odd IFF $H_1^{\sphericalangle}$ has already been studied in a different (less common) notation in a paper by Artru and Collins [9]. It is the asymmetry of present-day experimental interest regarding transversity. The extraction of $H_1^{\sphericalangle}$ from the process $e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X$ is the goal of a group [10] that will analyze the off-resonance data from the BELLE experiment at KEK. In the present article, we provide for the procedure of integrating and properly weighting the fully differential cross section to single out the relevant asymmetry. The extracted IFF will be of use to several ongoing or starting experiments aiming to measure transversity in the processes $e\,p^{\uparrow} \to (h_1h_2)X$ (HERMES, COMPASS) and $p\,p^{\uparrow} \to (h_1h_2)X$ (RHIC [10]).

However, the asymmetry involving $G_1^{\perp}$ also seems of experimental interest. It can be viewed as the chiral-even counterpart of the Artru-Collins asymmetry. An analogous asymmetry involving chiral-even fragmentation functions does not emerge when only one hadron is detected in each jet; this asymmetry is thus particular to the multi-hadron fragmentation case. But it can also be viewed as an asymmetry arising from a correlation between longitudinal jet handedness functions. As such it is relevant for single spin asymmetries with longitudinally polarized protons, $e\,\vec{p} \to (h_1h_2)X$ and $p\,\vec{p} \to (h_1h_2)X$, which are proportional to the well-known quark helicity distribution function g_1 (cf., e.g., Eq. (31) of Ref. [3]). Since g_1 is known to considerable accuracy, one can extract $G_1^{\perp}$ from $e\,\vec{p} \to (h_1h_2)X$ and actually predict our longitudinal jet handedness correlation in $e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X$, i.e. the expression given below in Eq. (38). Any experimental deviation may be related to a CP-violating effect of the QCD vacuum [11].

The function $G_1^{\perp}$ is also relevant for the studies of IFFs in the processes $e\,p^{\uparrow} \to (h_1h_2)X$ and $p\,p^{\uparrow} \to (h_1h_2)X$. There, next to the asymmetry proportional to the transversity function, another $G_1^{\perp}$ dependent asymmetry [7] occurs, which is proportional to the transverse momentum dependent distribution function g_{1T} [12]. This function (extrapolated to $x = 0$) gives information on violations of the Burkhardt-Cottingham sum rule. Apart from the intrinsic interest in such an asymmetry, it also shows the need for appropriate weight functions to separate the asymmetry proportional to $g_{1T}G_1^{\perp}$ from the asymmetries proportional to $h_1H_1^{\sphericalangle}$ and $h_1H_1^{\perp}$ (where h_1 denotes the transversity function).

The other results presented below, i.e. the other terms arising in the fully differential e^+e^- cross section, may also be of interest in the future and the notation used here hopefully will facilitate the communication between different experimental groups planning or performing two-hadron IFF-related studies for different processes.

The paper is organized as follows. In Sec. II we first discuss the kinematics of the process $e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X$. In Sec. III we present the cross section in terms of the interference fragmentation functions. Next, we investigate extensively the Artru-Collins azimuthal asymmetry (Sec. IV) and the newly-found longitudinal jet handedness asymmetry (Sec. V). During the discussion of these two asymmetries in $e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X$ we also remark on corresponding asymmetries in two-hadron inclusive DIS involving the same IFFs to facilitate comparison. We end with conclusions (Sec. VI).

II. KINEMATICS

We will consider the process $e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X$, schematically depicted in Fig. 1. An electron and a positron with momenta l and l', respectively, annihilate into a photon with timelike momentum $q = l + l'$ and $q^2 = Q^2$. A quark and an antiquark are then emitted and fragment each one into a residual jet and a pair of leading unpolarized hadrons (h_1, h_2) with momenta P_1, P_2, and masses M_1, M_2 (for the antiquark we have the corresponding notation $(\bar{h}_1, \bar{h}_2)$ with momenta $\overline{P}_1, \overline{P}_2$ and masses $\overline{M}_1, \overline{M}_2$). We introduce the vectors $P_h = P_1 + P_2$, $R = (P_1 - P_2)/2$, and $\overline{P}_h = \overline{P}_1 + \overline{P}_2$, $\overline{R} = (\overline{P}_1 - \overline{P}_2)/2$. The two jets are emitted in opposite directions, therefore $P_h \cdot \overline{P}_h \sim Q^2$. Neglecting $1/Q$ mass-term corrections [13], we can parametrize the momenta as

$$P_h^{\mu} \;=\; \frac{z_h Q}{\sqrt{2}}\, n_-^{\mu}$$

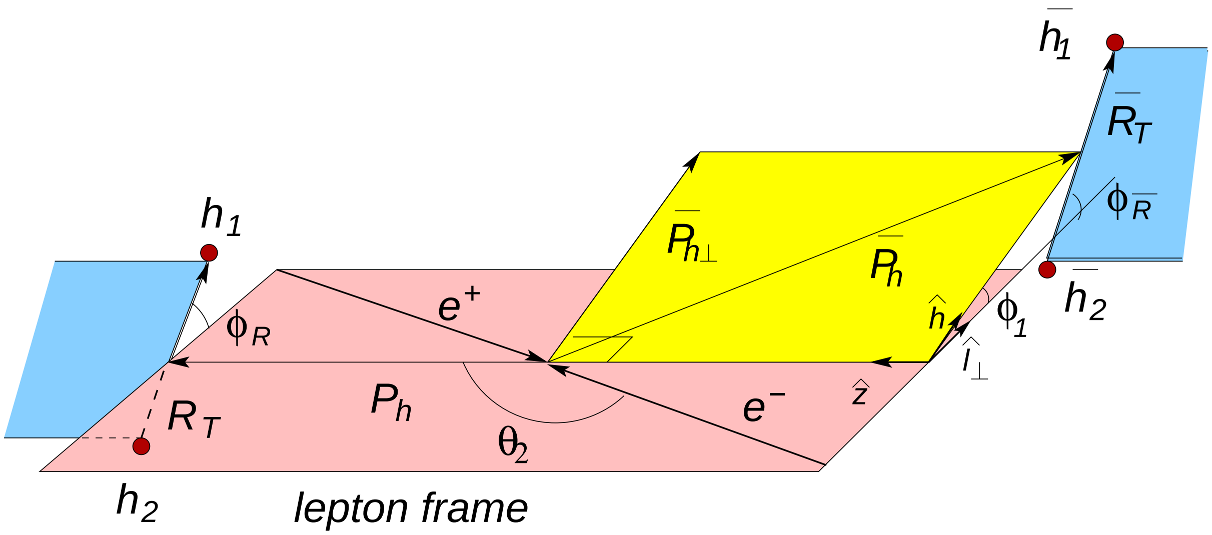

FIG. 1: Kinematic for the $e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)X$ process.

$$
\begin{aligned}
\overline{P}_h^\mu &= \frac{\overline{z}_h Q}{\sqrt{2}}\, n_+^\mu \\
q^\mu &= \frac{Q}{\sqrt{2}}\, n_-^\mu + \frac{Q}{\sqrt{2}}\, n_+^\mu + q_T^\mu \,,
\end{aligned} \tag{2}
$$

where $-q_T^2 \equiv Q_T^2 \ll Q^2$, and n_+, n_-, are light-like vectors satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$. We use the following notation, $a^\pm = a \cdot n_\mp$, for a generic 4-vector a with light-cone components $a = [a^-, a^+, \boldsymbol{a}_T]$. We define also $z = P_h^-/q^- \sim 2P_h \cdot q/Q^2 = z_h$, $\overline{z} = \overline{P}_h^+/q^+ \sim 2\overline{P}_h \cdot q/Q^2 = \overline{z}_h$, representing the light-cone momentum fractions of the fragmenting (anti-)quark carried by each hadron system. Analogously, we define the following fractions

$$
\begin{aligned}
\xi &= \frac{1}{2} + \frac{R^-}{P_h^-} = \frac{P_1^-}{P_1^- + P_2^-} \\
\overline{\xi} &= \frac{1}{2} + \frac{\overline{R}^-}{\overline{P}_h^-} = \frac{\overline{P}_1^-}{\overline{P}_1^- + \overline{P}_2^-} \,,
\end{aligned} \tag{3}
$$

that describe how the momentum of the (anti-)quark is split into each component of the hadron pair. The $\hat{z}$ axis is defined using P_h; in particular, from Fig. 1 it is $\boldsymbol{P}_h \parallel \hat{\boldsymbol{z}}$. It is useful to define the so-called $\perp$ plane [14] perpendicular to $\hat{\boldsymbol{z}}$, where $\boldsymbol{P}_{h\perp} = \boldsymbol{q}_\perp = 0$. Up to corrections in $Q_T^2/Q^2 \ll 1$, we have $\overline{P}_{h\perp}^\mu = -\overline{z} q_T^\mu$ and, consequently, $\overline{\boldsymbol{P}}_{h\perp} = -\overline{z}\boldsymbol{q}_T$. The $\perp$ plane is spanned by the two unit vectors

$$
\begin{aligned}
\hat{h} &= \frac{\overline{\boldsymbol{P}}_{h\perp}}{|\overline{\boldsymbol{P}}_{h\perp}|} = -\frac{\boldsymbol{q}_T}{|\boldsymbol{q}_T|} = (\cos\phi_1 \;,\; \sin\phi_1) \\
\hat{g}^i &= \epsilon_T^{ij}\hat{h}^j \equiv \epsilon^{-+ij}\hat{h}^j = \epsilon^{0ij3}\hat{h}^j = (\sin\phi_1 \;,\; -\cos\phi_1) \,,
\end{aligned} \tag{4}
$$

with ϕ_1 defined in Fig. 1. Therefore, we have $\hat{g} \cdot \boldsymbol{a} = (\boldsymbol{a} \times \hat{h})_z$ for a generic 3-vector $\boldsymbol{a}$. As in Deep Inelastic Scattering (DIS), the $\perp$ and transverse (T) components of a 4-vector can be obtained by the following tensors

$$
\begin{aligned}
g_T^{\mu\nu} &= g^{\mu\nu} - n_+^\mu n_-^\nu - n_-^\mu n_+^\nu \\
g_\perp^{\mu\nu} &= g_T^{\mu\nu} - \frac{\sqrt{2}(n_+^\mu q_T^\nu + n_+^\nu q_T^\mu)}{Q} \,.
\end{aligned} \tag{5}
$$

In the following, we will consistently neglect the $\mathcal{O}(1/Q)$ difference, thus not distinguishing between $\perp$ and T components of 4-vectors. From previous definitions we have also

$$
\begin{aligned}
|\boldsymbol{R}_T|^2 &= \xi(1-\xi)M_h^2 - (1-\xi)M_1^2 - \xi M_2^2 \\
|\overline{\boldsymbol{R}}_T|^2 &= \overline{\xi}(1-\overline{\xi})\overline{M}_h^2 - (1-\overline{\xi})\overline{M}_1^2 - \overline{\xi}\overline{M}_2^2 \,,
\end{aligned} \tag{6}
$$

where $P_h^2 = M_h^2$ and $\overline{P}_h^2 = \overline{M}_h^2$ are the invariant masses of the two final hadronic systems. The azimuthal angles $\phi_R, \phi_{\overline{R}}$, parametrizing the transverse components of $R, \overline{R}$, are depicted in Fig. 1. There, a further azimuthal angle ϕ^l should be considered which identifies the position of the lepton frame with respect to the laboratory frame. In Fig. 1 it is taken $\phi^l = 0$ for convenience, but in the following (see Secs. IV and V) we will have to retain its dependence explicitly such that the position of the hadron pairs with respect to the lepton frame is $\phi_R - \phi^l$ and $\phi_{\overline{R}} - \phi^l$, respectively.

The cross section for the 4-unpolarized-particle inclusive e^+e^- annihilation is

$$\frac{2P_1^0\, 2P_2^0\, 2\overline{P}_1^0\, 2\overline{P}_2^0\, d\sigma}{d\boldsymbol{P}_1 d\boldsymbol{P}_2 d\overline{\boldsymbol{P}}_1 d\overline{\boldsymbol{P}}_2} = \frac{\alpha^2}{Q^6}\, L_{\mu\nu} W^{\mu\nu}_{(4h)} \;, \tag{7}$$

where

$$\begin{aligned} L^{\mu\nu} &= Q^2 \Big[-2\, A(y)\, g_\perp^{\mu\nu} + 4\, B(y)\, \hat{z}^\mu \hat{z}^\nu - 4\, B(y) \left(\hat{l}_\perp^\mu \hat{l}_\perp^\nu + \frac{1}{2}\, g_\perp^{\mu\nu} \right) \\ &\quad - 2\, C(y) B^{1/2}(y) (\hat{z}^\mu \hat{l}_\perp^\nu + \hat{z}^\nu \hat{l}_\perp^\mu) \Big] \;, \end{aligned} \tag{8}$$

$$\begin{aligned} A(y) &= \left(\frac{1}{2} - y + y^2 \right) \stackrel{cm}{=} \frac{(1 + \cos^2 \theta_2)}{4} \\ B(y) &= y\,(1-y) \stackrel{cm}{=} \frac{\sin^2 \theta_2}{4} \\ C(y) &= 1 - 2y \stackrel{cm}{=} -\cos\theta_2 \;, \end{aligned} \tag{9}$$

is the lepton tensor. In fact, only the $L_\perp^{\mu\nu}$ part contributes at leading twist. The invariant $y = P_h \cdot l / P_h \cdot q \sim l^-/q^-$ becomes, in the lepton center-of-mass (cm) frame, $y = (1 + \cos\theta_2)/2$, where θ_2 is defined in Fig. 1. The unit vectors are defined as

$$\begin{aligned} \hat{l}_\perp^\mu &= l_\perp^\mu / |\boldsymbol{l}_\perp| \\ \hat{z}^\mu &= \frac{2}{zQ} P_h - \frac{q^\mu}{Q} \;. \end{aligned} \tag{10}$$

The hadronic tensor is

$$\begin{aligned} W^{\mu\nu}_{(4h)}(q; P_1, P_2, \overline{P}_1, \overline{P}_2) &= \frac{1}{(2\pi)^{10}} \sum\!\!\!\!\!\!\int_X \frac{d\boldsymbol{P}_X}{(2\pi)^3 2P_X^0}\, (2\pi)^4 \delta(q - P_X - P_1 - P_2 - \overline{P}_1 - \overline{P}_2) \\ &\quad \times \langle 0 | J^\mu(0) | P_X; P_1, P_2, \overline{P}_1, \overline{P}_2 \rangle \langle P_X; P_1, P_2, \overline{P}_1, \overline{P}_2 | J^\nu(0) | 0 \rangle \;. \end{aligned} \tag{11}$$

Such definition allows for recovering the corresponding formulae in the case of 2-particle, 1-particle and totally inclusive e^+e^- annihilation. For example, after integrating over one of the two hadrons in each pair,

$$\begin{aligned} \sum\!\!\!\!\!\!\int_{h_2} \frac{d\boldsymbol{P}_2}{2P_2^0} \sum\!\!\!\!\!\!\int_{\bar{h}_2} \frac{d\overline{\boldsymbol{P}}_2}{2\overline{P}_2^0} \frac{2P_1^0\, 2P_2^0\, 2\overline{P}_1^0\, 2\overline{P}_2^0 d\sigma}{d\boldsymbol{P}_1 d\boldsymbol{P}_2 d\overline{\boldsymbol{P}}_1 d\overline{\boldsymbol{P}}_2} &\equiv \frac{2P_1^0\, 2\overline{P}_1^0 d\sigma}{d\boldsymbol{P}_1 d\overline{\boldsymbol{P}}_1} = \frac{\alpha^2}{Q^6}\, L_{\mu\nu} \sum\!\!\!\!\!\!\int_{h_2} \frac{d\boldsymbol{P}_2}{2P_2^0} \sum\!\!\!\!\!\!\int_{\bar{h}_2} \frac{d\overline{\boldsymbol{P}}_2}{2\overline{P}_2^0}\, W^{\mu\nu}_{(4h)} \\ &= \frac{\alpha^2}{Q^6}\, L_{\mu\nu} \frac{1}{(2\pi)^4} \sum\!\!\!\!\!\!\int_X \frac{d\boldsymbol{P}_X}{(2\pi)^3 2P_X^0}\, (2\pi)^4 \delta(q - P_X - P_1 - P_2 - \overline{P}_1 - \overline{P}_2) \\ &\quad \times \sum\!\!\!\!\!\!\int_{h_2} \frac{d\boldsymbol{P}_2}{(2\pi)^3 2P_2^0} \sum\!\!\!\!\!\!\int_{\bar{h}_2} \frac{d\overline{\boldsymbol{P}}_2}{(2\pi)^3 2\overline{P}_2^0} \langle 0 | J^\mu | X, P_1, P_2, \overline{P}_1, \overline{P}_2 \rangle \langle X, P_1, P_2, \overline{P}_1, \overline{P}_2 | J^\nu | 0 \rangle \\ &= \frac{\alpha^2}{Q^6}\, L_{\mu\nu} \frac{1}{(2\pi)^4} \sum\!\!\!\!\!\!\int_{X'} \frac{d\boldsymbol{P}_{X'}}{(2\pi)^3 2P_{X'}^0}\, (2\pi)^4 \delta(q - P_{X'} - P_1 - \overline{P}_1) \sum\!\!\!\!\!\!\int_{h_2} \frac{d\boldsymbol{P}_2}{(2\pi)^3 2P_2^0} \\ &\quad \times \sum\!\!\!\!\!\!\int_{\bar{h}_2} \frac{d\overline{\boldsymbol{P}}_2}{(2\pi)^3 2\overline{P}_2^0} \langle 0 | J^\mu | X' - P_2 - \overline{P}_2, P_1, P_2, \overline{P}_1, \overline{P}_2 \rangle \langle X' - P_2 - \overline{P}_2, P_1, P_2, \overline{P}_1, \overline{P}_2 | J^\nu | 0 \rangle \\ &= \frac{\alpha^2}{Q^6}\, L_{\mu\nu} \frac{1}{(2\pi)^4} \sum\!\!\!\!\!\!\int_{X'} \frac{d\boldsymbol{P}_{X'}}{(2\pi)^3 2P_{X'}^0}\, (2\pi)^4 \delta(q - P_{X'} - P_1 - \overline{P}_1) \\ &\quad \times \langle 0 | J^\mu | X', P_1, \overline{P}_1 \rangle \langle X', P_1, \overline{P}_1 | J^\nu | 0 \rangle \\ &\equiv \frac{\alpha^2}{Q^6}\, L_{\mu\nu}\, W^{\mu\nu}_{(2h)} \;, \end{aligned} \tag{12}$$

we recover the cross section for the 2-particle inclusive e^+e^- annihilation [14] after the replacement $\overline{P}_1 \leftrightarrow P_2$. Further integrations over the detected hadrons lead to the 1-particle inclusive and totally inclusive cross sections (cf. [14]).

Consistently with Eq. (2), we have

$$
\begin{aligned}
P_h^+ \ll P_h^- &\rightarrow P_h^0 \sim \frac{1}{\sqrt{2}} P_h^- \\
R^+ \ll R^- &\rightarrow R^0 \sim \frac{1}{\sqrt{2}} R^- \\
\frac{R^-}{P_h^-} &= \xi - \frac{1}{2} \sim \frac{R^0}{P_h^0} \equiv \frac{E_R}{E_h} \\
\overline{P}_h^- \ll \overline{P}_h^+ &\rightarrow \overline{P}_h^0 \sim \frac{1}{\sqrt{2}} \overline{P}_h^+ \\
\overline{R}^- \ll \overline{R}^+ &\rightarrow \overline{R}^0 \sim \frac{1}{\sqrt{2}} \overline{R}^+ \\
\frac{\overline{R}^+}{\overline{P}_h^+} &\equiv \overline{\xi} - \frac{1}{2} \sim \frac{\overline{R}^0}{\overline{P}_h^0} \equiv \frac{\overline{E}_R}{\overline{E}_h} \, .
\end{aligned} \tag{13}
$$

The elementary phase space can then be transformed as follows:

$$
\begin{aligned}
\frac{d\boldsymbol{P}_1 d\boldsymbol{P}_2 d\overline{\boldsymbol{P}}_1 d\overline{\boldsymbol{P}}_2}{2P_1^0\, 2P_2^0\, 2\overline{P}_1^0\, 2\overline{P}_2^0} &= \frac{d\boldsymbol{P}_h\, d\boldsymbol{R}\, d\overline{\boldsymbol{P}}_h\, d\overline{\boldsymbol{R}}}{(E_h^2 - 4E_R^2)(\overline{E}_h^2 - 4\overline{E}_R^2)} \sim \frac{|\boldsymbol{P}_h|^2 d|\boldsymbol{P}_h| d\Omega_h\, d\boldsymbol{R}_T dR^-\, d\overline{\boldsymbol{P}}_{h\perp} d\overline{P}_h^+ d\overline{\boldsymbol{R}}_T d\overline{R}^+}{8\sqrt{2}\,(P_h^-)^2 \xi(1-\xi)(\overline{P}_h^+)^2 \overline{\xi}(1-\overline{\xi})} \\
&\sim \frac{zQ^2}{16\,\xi(1-\xi)}\, dz\, d\Omega_h\, d\boldsymbol{R}_T\, d\xi\, \frac{1}{4\overline{z}\overline{\xi}(1-\overline{\xi})} d\overline{\boldsymbol{P}}_{h\perp}\, d\overline{z}\, d\overline{\boldsymbol{R}}_T\, d\overline{\xi}\, ,
\end{aligned} \tag{14}
$$

where $d\Omega_h = 2dyd\phi^l$, since $\boldsymbol{P}_{h\perp} = 0$ and its azimuthal angle actually defines the position of the lepton plane with respect to the laboratory frame. Using Eq. (6) and

$$
\begin{aligned}
d\boldsymbol{R}_T &= J\, d\phi_R\, dM_h^2 \quad \text{with } J = \begin{vmatrix} \dfrac{\partial R_{Tx}}{\partial M_h^2} = \dfrac{\xi(1-\xi)}{2|\boldsymbol{R}_T|} \cos\phi_R & \dfrac{\partial R_{Tx}}{\partial \phi_R} = -|\boldsymbol{R}_T| \sin\phi_R \\ \dfrac{\partial R_{Ty}}{\partial M_h^2} = \dfrac{\xi(1-\xi)}{2|\boldsymbol{R}_T|} \sin\phi_R & \dfrac{\partial R_{Ty}}{\partial \phi_R} = |\boldsymbol{R}_T| \cos\phi_R \end{vmatrix} = \frac{\xi(1-\xi)}{2} \\
d\overline{\boldsymbol{R}}_T &= \frac{\overline{\xi}(1-\overline{\xi})}{2}\, d\phi_{\overline{R}}\, d\overline{M}_h^2\, ,
\end{aligned} \tag{15}
$$

the cross section can be rewritten as

$$
\frac{d\sigma}{d\boldsymbol{q}_T\, dz\, d\xi\, d\phi_R\, dM_h^2\, d\overline{z}\, d\overline{\xi}\, d\phi_{\overline{R}}\, d\overline{M}_h^2\, dy\, d\phi^l} = \frac{\alpha^2}{128\, Q^4}\, z\overline{z}\, L_{\mu\nu} W_{(4h)}^{\mu\nu} \, . \tag{16}
$$

III. CROSS SECTION

A. The hadronic tensor

To leading order the expression for the hadron tensor is

$$
W_{(4h)}^{\mu\nu} \sim 3\,(32)^2\, z\overline{z} \sum_{a,\overline{a}} e_a^2 \int d\boldsymbol{k}_T d\overline{\boldsymbol{k}}_T\, \delta^2(\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T - \boldsymbol{q}_T)\, \mathrm{Tr}\left[\left(\frac{1}{32\overline{z}} \int d\overline{k}^-\overline{\Delta} \right)_{\overline{k}^+ = \overline{P}_h^+/\overline{z}} \gamma^\mu \left(\frac{1}{32z} \int dk^+ \Delta \right)_{k^- = P_h^-/z} \gamma^\nu \right] . \tag{17}
$$

The (partly integrated) correlation function Δ is parametrized in terms of fragmentation functions as [7]

$$
\frac{1}{32z} \int dk^+ \Delta(k; P_h, R) \Big|_{k^- = P_h^-/z, \mathbf{k}_T}
$$

$$= \frac{1}{4\pi}\,\frac{1}{4}\left\{ D_1^a(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\,\not{n}_- - G_1^{\perp\,a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\,\frac{\epsilon_{\mu\nu\rho\sigma}\,\gamma^\mu\,n_-^\nu\,k_T^\rho\,R_T^\sigma}{M_1M_2}\,\gamma_5 \right.$$

$$\left. + H_1^{\sphericalangle\,a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\,\frac{\sigma_{\mu\nu}\,R_T^\mu\,n_-^\nu}{M_1+M_2} + H_1^{\perp\,a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\,\frac{\sigma_{\mu\nu}\,k_T^\mu\,n_-^\nu}{M_1+M_2}\right\}. \tag{18}$$

We parametrize the antiquark correlation function $\overline{\Delta}$ in the same way by employing overlined quantities, but in this case the suppressed $\overline{k}^-$ component is integrated over.

At leading twist, we have the usual nice probabilistic interpretation of the fragmentation functions in Eq. (18): D_1^a is the probability for an unpolarized quark with flavor a to fragment into the unpolarized hadron pair (h_1,h_2), $G_1^{\perp\,a}$ is the probability difference for a longitudinally polarized quark with flavor a and opposite chiralities to fragment into (h_1,h_2), both $H_1^{\sphericalangle\,a}$, $H_1^{\perp\,a}$, give the same probability difference but for a transversely polarized fragmenting quark. $G_1^{\perp\,a}$, $H_1^{\sphericalangle\,a}$, $H_1^{\perp\,a}$, are all *naive* T-odd and $H_1^{\sphericalangle\,a}$, $H_1^{\perp\,a}$ are furthermore chiral odd. The function $H_1^{\perp\,a}$ represents a generalization of the Collins effect, namely for two hadrons instead of one. However, $H_1^{\sphericalangle\,a}$ originates from a genuinely new effect, because it relates the transverse polarization of the fragmenting quark to the angular distribution of the hadron pair in the $\perp$ plane (defined in Sec. II [7]).

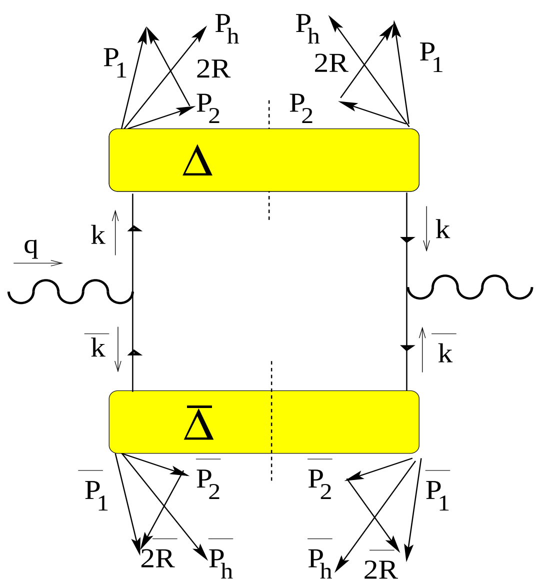

FIG. 2: Leading-twist contribution to 4p-inclusive e^+e^- annihilation.

B. The fully differential cross section

For the case of the e^+e^- annihilation into four unpolarized (or spinless) hadrons with two leading hadrons in each current jet (see Fig. 2 for a diagrammatic representation at leading order), the differential cross section at leading order in $1/Q$ and α_s is (including now summation over flavor indices with quark charges e_a in units of e)

$$\frac{d\sigma(e^+e^-\to(h_1h_2)(\bar h_1\bar h_2)X)}{d\boldsymbol{q}_T\,dz\,d\xi\,dM_h^2\,d\phi_R\,d\overline{z}\,d\overline{\xi}\,d\overline{M}_h^2\,d\phi_{\overline{R}}\,dy\,d\phi^l} = \sum_{a,\overline{a}} e_a^2\,\frac{6\,\alpha^2}{Q^2}\,z^2\overline{z}^2\left\{A(y)\,\mathcal{F}\left[D_1^a\overline{D}_1^a\right]\right.$$

$$+\cos(2\phi_1)\,B(y)\,\mathcal{F}\left[\left(2\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T - \boldsymbol{k}_T\cdot\overline{\boldsymbol{k}}_T\right)\frac{H_1^{\perp a}\overline{H}_1^{\perp a}}{(M_1+M_2)(\overline{M}_1+\overline{M}_2)}\right]$$

$$-\sin(2\phi_1)\,B(y)\,\mathcal{F}\left[\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T + \hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\,\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\right)\frac{H_1^{\perp a}\overline{H}_1^{\perp a}}{(M_1+M_2)(\overline{M}_1+\overline{M}_2)}\right]$$

$$+\cos(\phi_R+\phi_{\overline{R}}-2\phi^l)\; B(y)\; |\boldsymbol{R}_T|\, |\overline{\boldsymbol{R}}_T|\; \mathcal{F}\left[\frac{H_1^{\sphericalangle a}\overline{H}_1^{\sphericalangle a}}{(M_1+M_2)(\overline{M}_1+\overline{M}_2)}\right]$$

$$+\cos(\phi_1+\phi_R-\phi^l)\; B(y)\; |\boldsymbol{R}_T|\; \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\, \frac{H_1^{\sphericalangle a}\overline{H}_1^{\perp a}}{(M_1+M_2)(\overline{M}_1+\overline{M}_2)}\right]$$

$$-\sin(\phi_1+\phi_R-\phi^l)\; B(y)\; |\boldsymbol{R}_T|\; \mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\, \frac{H_1^{\sphericalangle a}\overline{H}_1^{\perp a}}{(M_1+M_2)(\overline{M}_1+\overline{M}_2)}\right]$$

$$+\cos(\phi_1+\phi_{\overline{R}}-\phi^l)\; B(y)\; |\overline{\boldsymbol{R}}_T|\; \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\, \frac{H_1^{\perp a}\overline{H}_1^{\sphericalangle a}}{(M_1+M_2)(\overline{M}_1+\overline{M}_2)}\right]$$

$$-\sin(\phi_1+\phi_{\overline{R}}-\phi^l)\; B(y)\; |\overline{\boldsymbol{R}}_T|\; \mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\, \frac{H_1^{\perp a}\overline{H}_1^{\sphericalangle a}}{(M_1+M_2)(\overline{M}_1+\overline{M}_2)}\right]$$

$$+A(y)\; |\boldsymbol{R}_T|\, |\overline{\boldsymbol{R}}_T| \left[\sin(\phi_1-\phi_R+\phi^l)\,\sin(\phi_1-\phi_{\overline{R}}+\phi^l)\; \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\, \frac{G_1^{\perp a}\overline{G}_1^{\perp a}}{M_1M_2\overline{M}_1\overline{M}_2}\right]\right.$$

$$+\sin(\phi_1-\phi_R+\phi^l)\,\cos(\phi_1-\phi_{\overline{R}}+\phi^l)\; \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\, \frac{G_1^{\perp a}\overline{G}_1^{\perp a}}{M_1M_2\overline{M}_1\overline{M}_2}\right]$$

$$+\cos(\phi_1-\phi_R+\phi^l)\,\sin(\phi_1-\phi_{\overline{R}}+\phi^l)\; \mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\, \frac{G_1^{\perp a}\overline{G}_1^{\perp a}}{M_1M_2\overline{M}_1\overline{M}_2}\right]$$

$$\left.\left.+\cos(\phi_1-\phi_R+\phi^l)\,\cos(\phi_1-\phi_{\overline{R}}+\phi^l)\; \mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\, \frac{G_1^{\perp a}\overline{G}_1^{\perp a}}{M_1M_2\overline{M}_1\overline{M}_2}\right]\right]\right\}, \tag{19}$$

where the convolution $\mathcal{F}$ is defined as

$$\mathcal{F}\left[w(\boldsymbol{k}_T,\overline{\boldsymbol{k}}_T)\; D^a\overline{D}^a\right] \equiv \int d\boldsymbol{k}_T\; d\overline{\boldsymbol{k}}_T\; \delta^2(\overline{\boldsymbol{k}}_T+\boldsymbol{k}_T-\boldsymbol{q}_T)\; w(\boldsymbol{k}_T,\overline{\boldsymbol{k}}_T)\; D^a(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

$$\overline{D}^a(\overline{z},\overline{\xi},\overline{\boldsymbol{k}}_T^2,\overline{\boldsymbol{R}}_T^2,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T)\,. \tag{20}$$

The azimuthal dependence is dictated by the fact that any information about the azimuthal asymmetry of the distribution of the four hadrons must be encoded by the relative position of $\boldsymbol{R}_T$ and $\overline{\boldsymbol{R}}_T$ with respect to the lepton frame, i.e. by the $\phi_R-\phi^l$ and $\phi_{\overline{R}}-\phi^l$ angles, respectively, and by the azimuthal position of the lepton frame itself.

IV. THE ARTRU-COLLINS AZIMUTHAL ASYMMETRY

In this Section, we will obtain an azimuthal asymmetry in the distribution of the four hadrons that arises only due to the transverse relative momenta of each pair, i.e. only due to the relative position of each pair plane with respect to the lepton plane (see Fig. 1). For this purpose, the cross section of Eq. (19) must be properly weighted and its dependence on the intrinsic transverse momenta of the quarks integrated out. We present the procedure in considerable detail, since this will form a crucial aspect of a practical analysis of experimental data. We will show that only $H_1^{\sphericalangle}$ survives the integration, which is the same fragmentation function appearing in the single-spin asymmetry that can be built at leading twist in the case of two-hadron inclusive DIS [5, 6, 7]. Therefore, under the hypothesis of factorization (collinear factorization in this particular case), the combined analysis of the two semi-inclusive processes allows in principle to deduce the fragmentation function from e^+e^- and then disentangle the transversity distribution in the corresponding DIS cross section at leading twist.

We define the asymmetry

$$A(y,z,\overline{z},M_h^2,\overline{M}_h^2) = \frac{\langle\cos(\phi_R+\phi_{\overline{R}}-2\phi^l)\rangle}{\langle 1\rangle}$$

$$
\equiv \left[\int d\xi \int d\overline{\xi} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \cos(\phi_R + \phi_{\overline{R}} - 2\phi^l) \right.
$$

$$
\left. \times \int d\boldsymbol{q}_T \, \frac{d\sigma(e^+e^- \to (h_1 h_2)(\overline{h}_1 \overline{h}_2) X)}{dy\, d\phi^l\, dz\, d\overline{z}\, d\xi\, d\overline{\xi}\, d\boldsymbol{q}_T\, dM_h^2\, d\phi_R\, d\overline{M}_h^2\, d\phi_{\overline{R}}} \right]
$$

$$
\times \left[\int d\xi \int d\overline{\xi} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \right.
$$

$$
\left. \times \int d\boldsymbol{q}_T \, \frac{d\sigma(e^+e^- \to (h_1 h_2)(\overline{h}_1 \overline{h}_2) X)}{dy\, d\phi^l\, dz\, d\overline{z}\, d\xi\, d\overline{\xi}\, d\boldsymbol{q}_T\, dM_h^2\, d\phi_R\, d\overline{M}_h^2\, d\phi_{\overline{R}}} \right]^{-1} . \qquad (21)
$$

Let us consider first the term in the numerator, involving the trigonometric weight. If we insert the cross section of Eq. (19) into it and consider just the average over the azimuthal positions ϕ^l of the lepton plane, we can see that the first three terms give a vanishing contribution because they involve the integral

$$
\int_0^{2\pi} \frac{d\phi^l}{2\pi} \cos(\phi_R + \phi_{\overline{R}} - 2\phi^l) = 0 \, . \qquad (22)
$$

The fourth term gives

$$
\sum_{a,\overline{a}} e_a 2 \, \frac{6\,\alpha^2}{Q^2} \, z^2 \overline{z}^2 \, B(y) \int d\xi \int d\overline{\xi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \, |\boldsymbol{R}_T| \, |\overline{\boldsymbol{R}}_T| \, \cos^2(\phi_R + \phi_{\overline{R}} - 2\phi^l) \int d\boldsymbol{q}_T
$$

$$
\times \mathcal{F} \left[\frac{H_1^{\sphericalangle a} \overline{H}_1^{\sphericalangle a}}{(M_1 + M_2)(\overline{M}_1 + \overline{M}_2)} \right] =
$$

$$
\sum_{a,\overline{a}} e_a^2 \, \frac{6\,\alpha^2}{Q^2} \, z^2 \overline{z}^2 \, B(y) \int d\xi \int d\overline{\xi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \, \frac{|\boldsymbol{R}_T| \, |\overline{\boldsymbol{R}}_T|}{2(M_1 + M_2)(\overline{M}_1 + \overline{M}_2)} \int d\boldsymbol{q}_T
$$

$$
\times \int d\boldsymbol{k}_T \, d\overline{\boldsymbol{k}}_T \, \delta^2(\overline{\boldsymbol{k}}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) \, H_1^{\sphericalangle a}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T) \, \overline{H}_1^{\sphericalangle a}(\overline{z}, \overline{\xi}, \overline{\boldsymbol{k}}_T^2, \overline{\boldsymbol{R}}_T^2, \overline{\boldsymbol{k}}_T \cdot \overline{\boldsymbol{R}}_T) =
$$

$$
\sum_{a,\overline{a}} e_a^2 \, \frac{3\alpha^2}{Q^2} \, z^2 \overline{z}^2 \, \frac{B(y)}{(M_1 + M_2)(\overline{M}_1 + \overline{M}_2)} H_{1\,(R)}^{\sphericalangle a}(z, M_h^2) \, \overline{H}_{1\,(R)}^{\sphericalangle a}(\overline{z}, \overline{M}_h^2) \, , \qquad (23)
$$

where

$$
\begin{aligned}
H_{1\,(R)}^{\sphericalangle a}(z, M_h^2) &= \int d\xi \, |\boldsymbol{R}_T| \int_0^{2\pi} d\phi_R \int d\boldsymbol{k}_T \, H_1^{\sphericalangle a}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T) \\
\overline{H}_{1\,(R)}^{\sphericalangle a}(\overline{z}, \overline{M}_h^2) &= \int d\overline{\xi} \, |\overline{\boldsymbol{R}}_T| \int_0^{2\pi} d\phi_{\overline{R}} \int d\overline{\boldsymbol{k}}_T \, \overline{H}_1^{\sphericalangle a}(\overline{z}, \overline{\xi}, \overline{\boldsymbol{k}}_T^2, \overline{\boldsymbol{R}}_T^2, \overline{\boldsymbol{k}}_T \cdot \overline{\boldsymbol{R}}_T)
\end{aligned} \qquad (24)
$$

are the same moments of fragmentation functions that appear in the following leading twist single-spin asymmetry arising in two-hadron semi-inclusive DIS off a transversely polarized target (see Eq. (17) of Ref. [7]):

$$
\langle \sin(\phi_R - 2\phi^l) \rangle \propto B(y) \, |\boldsymbol{S}_\perp| \, \sum_a e_a^2 \, x \, h_1^a(x) \, H_{1\,(R)}^{\sphericalangle a}(z, M_h^2) \, , \qquad (25)
$$

where $\boldsymbol{S}_\perp$ is the transverse polarization of the target and x is the light-cone momentum fraction of the quark.

The fifth through eighth terms give again a vanishing contribution, because they involve integrals of the kind

$$
\int_0^{2\pi} \frac{d\phi^l}{2\pi} \left\{ \begin{array}{c} \cos 2\phi^l \\ \sin 2\phi^l \end{array} \right\} \otimes \left\{ \begin{array}{c} \cos \phi^l \\ \sin \phi^l \end{array} \right\} = 0 \, . \qquad (26)
$$

Finally, it is instructive to check that the last four terms in Eq. (19) vanish only after the combined effect of the average over $d\phi^l$ and the integral upon $d\boldsymbol{q}_T$. In fact,

$$\sum_{a,\overline{a}} e_a^2 \frac{6\,\alpha^2}{Q^2}\, z^2\overline{z}^2 \quad A(y) \int d\xi \int d\overline{\xi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\overline{\phi}_R \int_0^{2\pi} \frac{d\phi^l}{2\pi} \frac{|\boldsymbol{R}_T|\,|\overline{\boldsymbol{R}}_T|}{M_1 M_2 \overline{M}_1 \overline{M}_2} \cos(\phi_R + \phi_{\overline{R}} - 2\phi^l) \int d\boldsymbol{q}_T$$

$$\times \Big\{ \sin(\phi_1 - \phi_R + \phi^l)\, \sin(\phi_1 - \phi_{\overline{R}} + \phi^l)\, \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\, G_1^{\perp a}\overline{G}_1^{\perp a}\right] +$$

$$\sin(\phi_1 - \phi_R + \phi^l)\, \cos(\phi_1 - \phi_{\overline{R}} + \phi^l)\, \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\, G_1^{\perp a}\overline{G}_1^{\perp a}\right] +$$

$$\cos(\phi_1 - \phi_R + \phi^l)\, \sin(\phi_1 - \phi_{\overline{R}} + \phi^l)\, \mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\, G_1^{\perp a}\overline{G}_1^{\perp a}\right] +$$

$$\cos(\phi_1 - \phi_R + \phi^l)\, \cos(\phi_1 - \phi_{\overline{R}} + \phi^l)\, \mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\, G_1^{\perp a}\overline{G}_1^{\perp a}\right] \Big\} =$$

$$\sum_{a,\overline{a}} e_a^2 \frac{6\,\alpha^2}{Q^2}\, z^2\overline{z}^2 \quad A(y) \int d\xi \int d\overline{\xi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \frac{|\boldsymbol{R}_T|\,|\overline{\boldsymbol{R}}_T|}{M_1 M_2 \overline{M}_1 \overline{M}_2} \int d\boldsymbol{q}_T \int d\boldsymbol{k}_T\, d\overline{\boldsymbol{k}}_T\, \delta^2(\overline{\boldsymbol{k}}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)$$

$$\times \Big\{ \frac{1}{4}(\cos^2\phi_1 - \sin^2\phi_1)\, (\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T - \hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T) +$$

$$\frac{1}{4} \sin\phi_1\, \cos\phi_1\, (\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T + \hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T) \Big\}\; G_1^{\perp a}\overline{G}_1^{\perp a} =$$

$$\sum_{a,\overline{a}} e_a^2 \frac{3\,\alpha^2}{2Q^2}\, z^2\overline{z}^2 \quad A(y) \int d\xi \int d\overline{\xi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \frac{|\boldsymbol{R}_T|\,|\overline{\boldsymbol{R}}_T|}{M_1 M_2 \overline{M}_1 \overline{M}_2} \int d\boldsymbol{k}_T\, d\overline{\boldsymbol{k}}_T$$

$$\times \Big\{ \frac{(\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T)_y^2 - (\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T)_x^2}{|\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T|^4} \left[(\boldsymbol{k}_T \times \overline{\boldsymbol{k}}_T)_z^2 + \boldsymbol{k}_T \cdot (\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T)\, \overline{\boldsymbol{k}}_T \cdot (\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T)\right] +$$

$$2\, \frac{(\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T)_x\, (\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T)_y}{|\boldsymbol{k}_T + \overline{\boldsymbol{k}}_T|^4} (\boldsymbol{k}_T \times \overline{\boldsymbol{k}}_T)_z\, (\overline{\boldsymbol{k}}_T^2 - \boldsymbol{k}_T^2) \Big\}$$

$$\times G_1^{\perp a}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)\, \overline{G}_1^{\perp a}(\overline{z}, \overline{\xi}, \overline{\boldsymbol{k}}_T^2, \overline{\boldsymbol{R}}_T^2, \overline{\boldsymbol{k}}_T \cdot \overline{\boldsymbol{R}}_T) = 0\,, \tag{27}$$

because the last two integrands are odd under the transformations

$$k_{T\,x} \leftrightarrow k_{T\,y} \quad , \quad \overline{k}_{T\,x} \leftrightarrow \overline{k}_{T\,y}$$
$$R_{T\,x} \leftrightarrow R_{T\,y} \quad , \quad \overline{R}_{T\,x} \leftrightarrow \overline{R}_{T\,y}\,. \tag{28}$$

Hence, we have

$$\begin{aligned} \langle \cos(\phi_R + \phi_{\overline{R}} - 2\phi^l) \rangle &= \int d\xi \int d\overline{\xi} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \cos(\phi_R + \phi_{\overline{R}} - 2\phi^l) \\ &\quad \times \int d\boldsymbol{q}_T \frac{d\sigma(e^+e^- \to (h_1 h_2)(\overline{h}_1\overline{h}_2)X)}{dy\, d\phi^l\, dz\, d\overline{z}\, d\xi\, d\overline{\xi}\, d\boldsymbol{q}_T\, dM_h^2\, d\phi_R\, d\overline{M}_h^2\, d\phi_{\overline{R}}} \\ &= \sum_{a,\overline{a}} e_a^2 \frac{3\alpha^2}{Q^2} \frac{z^2\overline{z}^2\, B(y)}{(M_1 + M_2)(\overline{M}_1 + \overline{M}_2)} H_{1\,(R)}^{\sphericalangle\, a}(z, M_h^2)\, \overline{H}_{1\,(R)}^{\sphericalangle\, a}(\overline{z}, \overline{M}_h^2)\,. \end{aligned} \tag{29}$$

In a similar way, it is straightforward to prove that the unweighted cross section receives a contribution only from the first term of Eq. (19), i.e.

$$\langle 1 \rangle = \int d\xi \int d\overline{\xi} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}}$$

$$\times \int d\boldsymbol{q}_T \, \frac{d\sigma(e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)X)}{dy\, d\phi^l\, dz\, d\overline{z}\, d\xi\, d\overline{\xi}\, d\boldsymbol{q}_T\, dM_h^2\, d\phi_R\, d\overline{M}_h^2\, d\phi_{\overline{R}}}$$

$$= \sum_{a,\overline{a}} e_a^2 \frac{6\alpha^2}{Q^2} A(y)\, z^2\, \overline{z}^2\, D_1^a(z, M_h^2)\, \overline{D}_1^a(\overline{z}, \overline{M}_h^2)\,, \tag{30}$$

where

$$\begin{aligned} D_1^a(z, M_h^2) &= \int d\xi \int_0^{2\pi} d\phi_R \int d\boldsymbol{k}_T\, D_1^a(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T) \\ \overline{D}_1^a(\overline{z}, \overline{M}_h^2) &= \int d\overline{\xi} \int_0^{2\pi} d\phi_{\overline{R}} \int d\overline{\boldsymbol{k}}_T\, \overline{D}_1^a(\overline{z}, \overline{\xi}, \overline{\boldsymbol{k}}_T^2, \overline{\boldsymbol{R}}_T^2, \overline{\boldsymbol{k}}_T \cdot \overline{\boldsymbol{R}}_T) \end{aligned} \tag{31}$$

are the same fragmentation functions as arising in the unweighted cross section at leading twist for the two-hadron semi-inclusive DIS process (see Eq. (18) of Ref. [7]).

The final expression for the azimuthal asymmetry is, from Eq. (21),

$$\begin{aligned} A(y, z, \overline{z}, M_h^2, \overline{M}_h^2) &= \frac{1}{2} \left[\sum_{a,\overline{a}} e_a^2 \frac{z^2 \overline{z}^2\, B(y)}{(M_1 + M_2)(\overline{M}_1 + \overline{M}_2)} H_{1\,(R)}^{\sphericalangle\, a}(z, M_h^2)\, \overline{H}_{1\,(R)}^{\sphericalangle\, a}(\overline{z}, \overline{M}_h^2) \right] \\ &\quad \times \left[\sum_{a,\overline{a}} e_a^2\, z^2\, \overline{z}^2\, A(y)\, D_1^a(z, M_h^2)\, \overline{D}_1^a(\overline{z}, \overline{M}_h^2) \right]^{-1} . \end{aligned} \tag{32}$$

This azimuthal asymmetry is our version of the Artru-Collins asymmetry [9].

V. THE LONGITUDINAL JET HANDEDNESS AZIMUTHAL ASYMMETRY

The other azimuthal asymmetry we will explicitly derive is defined as

$$\begin{aligned} A(y, z, \overline{z}, M_h^2, \overline{M}_h^2) &= \frac{\langle \cos(2(\phi_R - \phi_{\overline{R}})) \rangle}{\langle 1 \rangle} \\ &\equiv \Bigg[\int d\xi \int d\overline{\xi} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}}\, \cos(2(\phi_R - \phi_{\overline{R}})) \\ &\qquad \times \int d\boldsymbol{q}_T \, \frac{d\sigma(e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)X)}{dy\, d\phi^l\, dz\, d\overline{z}\, d\xi\, d\overline{\xi}\, d\boldsymbol{q}_T\, dM_h^2\, d\phi_R\, d\overline{M}_h^2\, d\phi_{\overline{R}}} \Bigg] \\ &\quad \times \Bigg[\int d\xi \int d\overline{\xi} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \\ &\qquad \times \int d\boldsymbol{q}_T \, \frac{d\sigma(e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)X)}{dy\, d\phi^l\, dz\, d\overline{z}\, d\xi\, d\overline{\xi}\, d\boldsymbol{q}_T\, dM_h^2\, d\phi_R\, d\overline{M}_h^2\, d\phi_{\overline{R}}} \Bigg]^{-1} . \end{aligned} \tag{33}$$

Note that this asymmetry is independent of the orientation of the lepton scattering plane, contrary to the asymmetry of the previous Section.

By performing the integrations in the same order as in Eqs. (22-27), the surviving terms are

$$\begin{aligned} &\sum_{a,\overline{a}} e_a^2 \frac{6\,\alpha^2}{Q^2} z^2 \overline{z}^2\ A(y) \int d\xi \int d\overline{\xi} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_{\overline{R}} \int_0^{2\pi} \frac{d\phi^l}{2\pi} \frac{|\boldsymbol{R}_T|\, |\overline{\boldsymbol{R}}_T|}{M_1 M_2 \overline{M}_1 \overline{M}_2} \cos(2(\phi_R - \phi_{\overline{R}})) \int d\boldsymbol{q}_T \\ &\qquad \times \Big\{ \sin(\phi_1 - \phi_R + \phi^l)\, \sin(\phi_1 - \phi_{\overline{R}} + \phi^l)\, \mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\, \hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\, G_1^{\perp a} \overline{G}_1^{\perp a} \right] + \end{aligned}$$

$$
\begin{aligned}
&\sin(\phi_1-\phi_R+\phi^l)\,\cos(\phi_1-\phi_{\overline{R}}+\phi^l)\,\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\,G_1^{\perp a}\overline{G}_1^{\perp a}\right]+\\
&\cos(\phi_1-\phi_R+\phi^l)\,\sin(\phi_1-\phi_{\overline{R}}+\phi^l)\,\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\,G_1^{\perp a}\overline{G}_1^{\perp a}\right]+\\
&\left.\cos(\phi_1-\phi_R+\phi^l)\,\cos(\phi_1-\phi_{\overline{R}}+\phi^l)\,\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\,G_1^{\perp a}\overline{G}_1^{\perp a}\right]\right\}=
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{a,\overline{a}} e_a^2\,\frac{6\,\alpha^2}{Q^2}\,z^2\overline{z}^2\quad A(y)\;\int d\xi\int d\overline{\xi}\int_0^{2\pi}d\phi_R\int_0^{2\pi}d\phi_{\overline{R}}\,\frac{1}{2M_1M_2\overline{M}_1\overline{M}_2}\;\cos(2(\phi_R-\phi_{\overline{R}}))\\
&\quad\times\left\{\cos(\phi_R-\phi_{\overline{R}})\,\hat{\boldsymbol{R}}_T\cdot\hat{\overline{\boldsymbol{R}}}_T\int d\boldsymbol{k}_T\,\boldsymbol{k}_T\cdot\boldsymbol{R}_T\,G_1^{\perp a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\int d\overline{\boldsymbol{k}}_T\,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T\overline{G}_1^{\perp a}(\overline{z},\overline{\xi},\overline{\boldsymbol{k}}_T^2,\overline{\boldsymbol{R}}_T^2,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T)\right.\\
&\quad\left.+\sin(\phi_R-\phi_{\overline{R}})\,\hat{\boldsymbol{R}}_T\times\hat{\overline{\boldsymbol{R}}}_T\int d\boldsymbol{k}_T\,\boldsymbol{k}_T\cdot\boldsymbol{R}_T\,G_1^{\perp a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\int d\overline{\boldsymbol{k}}_T\,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T\overline{G}_1^{\perp a}(\overline{z},\overline{\xi},\overline{\boldsymbol{k}}_T^2,\overline{\boldsymbol{R}}_T^2,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T)\right\}\\
&=\sum_{a,\overline{a}} e_a^2\,\frac{3\,\alpha^2}{2Q^2}\,z^2\overline{z}^2\quad A(y)\;\frac{1}{M_1M_2\overline{M}_1\overline{M}_2}\,G_1^{\perp a}(z,M_h^2)\,\overline{G}_1^{\perp a}(\overline{z},\overline{M}_h^2)\;, \qquad (34)
\end{aligned}
$$

where

$$
\begin{aligned}
G_1^{\perp a}(z,M_h^2)\;&\equiv\;\int d\xi\int_0^{2\pi}d\phi_R\int d\boldsymbol{k}_T\,\boldsymbol{k}_T\cdot\boldsymbol{R}_T\,G_1^{\perp a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\\
\overline{G}_1^{\perp a}(\overline{z},\overline{M}_h^2)\;&\equiv\;\int d\overline{\xi}\int_0^{2\pi}d\phi_{\overline{R}}\int d\overline{\boldsymbol{k}}_T\,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T\,\overline{G}_1^{\perp a}(\overline{z},\overline{\xi},\overline{\boldsymbol{k}}_T^2,\overline{\boldsymbol{R}}_T^2,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T)\;, \qquad (35)
\end{aligned}
$$

are weighted moments of the same IFF that appears in the cross section at leading twist for two-hadron semi-inclusive DIS off a transversely polarized target (see Eq. (10) of Ref. [7]). For simplicity of notation, these moments carry no further subscripts, as opposed to Eq. (24).

In Eq. (34), the first step follows from first integrating over $d\phi^l$, which implies the disappearance of the explicit ϕ_1 dependence, and by performing the $d\boldsymbol{q}_T$ integration using identities like

$$
\begin{aligned}
\int d\boldsymbol{q}_T\,\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\,+\,\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\right]\,G_1^{\perp a}\overline{G}_1^{\perp a}\;&=\;\int d\boldsymbol{k}_T\,k_T^i\,G_1^{\perp a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\\
&\qquad\times\int d\overline{\boldsymbol{k}}_T\,\overline{k}_T^i\,\overline{G}_1^{\perp a}(\overline{z},\overline{\xi},\overline{\boldsymbol{k}}_T^2,\overline{\boldsymbol{R}}_T^2,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T)\\
\int d\boldsymbol{q}_T\,\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\overline{\boldsymbol{k}}_T\,-\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{g}}\cdot\overline{\boldsymbol{k}}_T\right]\,G_1^{\perp a}\overline{G}_1^{\perp a}\;&=\;\epsilon_{3ij}\int d\boldsymbol{k}_T\,k_T^i\,G_1^{\perp a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\\
&\qquad\times\int d\overline{\boldsymbol{k}}_T\,\overline{k}_T^j\,\overline{G}_1^{\perp a}(\overline{z},\overline{\xi},\overline{\boldsymbol{k}}_T^2,\overline{\boldsymbol{R}}_T^2,\overline{\boldsymbol{k}}_T\cdot\overline{\boldsymbol{R}}_T)\;. \qquad (36)
\end{aligned}
$$

The latter integrations can only result in a function of $(z,\overline{z},\xi,\overline{\xi},\boldsymbol{R}_T^2,\overline{\boldsymbol{R}}_T^2)$ multiplying the products $\hat{\boldsymbol{R}}_T\cdot\hat{\overline{\boldsymbol{R}}}_T$ and $\hat{\boldsymbol{R}}_T\times\hat{\overline{\boldsymbol{R}}}_T$ of unit vectors, respectively, since there are no other available vectors.

From Eq. (34) it is also easy to check that $G_1^\perp$ does not enter in the integrated, unweighted, cross section. The resulting expression for the numerator in Eq. (33) becomes

$$
\begin{aligned}
\langle\cos(2(\phi_R-\phi_{\overline{R}}))\rangle\;&=\;\int d\xi\int d\overline{\xi}\int_0^{2\pi}\frac{d\phi^l}{2\pi}\int_0^{2\pi}d\phi_R\int_0^{2\pi}d\phi_{\overline{R}}\,\cos(\phi_R-\phi_{\overline{R}})\\
&\qquad\times\int d\boldsymbol{q}_T\,\frac{d\sigma(e^+e^-\to(h_1h_2)(\bar{h}_1\bar{h}_2)X)}{dy\,d\phi^l\,dz\,d\overline{z}\,d\xi\,d\overline{\xi}\,d\boldsymbol{q}_T\,dM_h^2\,d\phi_R\,d\overline{M}_h^2\,d\phi_{\overline{R}}}\\
&=\;\sum_{a,\overline{a}} e_a^2\,\frac{3\,\alpha^2}{2Q^2}\,z^2\overline{z}^2\,A(y)\,\frac{1}{M_1M_2\overline{M}_1\overline{M}_2}\,G_1^{\perp a}(z,M_h^2)\,\overline{G}_1^{\perp a}(\overline{z},\overline{M}_h^2)\;. \qquad (37)
\end{aligned}
$$

The final expression for Eq. (33) is

$$A(y,z,\overline{z},M_h^2,\overline{M}_h^2) = \frac{1}{4}\left[\sum_{a,\overline{a}} e_a^2 \frac{z^2\overline{z}^2}{M_1M_2\overline{M}_1\overline{M}_2} G_1^{\perp\,a}(z,M_h^2)\,\overline{G}_1^{\perp\,a}(\overline{z},\overline{M}_h^2)\right] \times \left[\sum_{a,\overline{a}} e_a^2\, z^2\,\overline{z}^2\, D_1^a(z,M_h^2)\,\overline{D}_1^a(\overline{z},\overline{M}_h^2)\right]^{-1} . \tag{38}$$

It is possible to consider the $\boldsymbol{q}_T^2$ weighting and get $\boldsymbol{k}_T^2$ moments, but we will not do so here. Rather, it is important to remark that the weighting factor $\boldsymbol{k}_T\cdot\boldsymbol{R}_T$ in Eq. (35) is crucial, since the function

$$\int d\xi \int_0^{2\pi} d\phi_R \int d\boldsymbol{k}_T\; G_1^{\perp a}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) \tag{39}$$

does not occur due to parity invariance [3, 4, 15]. Nevertheless, the chiral-even IFF $G_1^\perp$ can provide a probe of g_1 as it emerges from the expression of the cross section at leading twist for the two-hadron semi-inclusive DIS off a longitudinally polarized target (see Eq. (31) of Ref. [3]):

$$\frac{d\sigma(e\vec{p}\to e'h_1h_2X)_{OL}}{d\Omega\,dx\,dz\,d\xi\,d\boldsymbol{P}_{h\perp}\,d\boldsymbol{R}_T} \propto \left\{\ldots - \lambda\,|\boldsymbol{R}_T|\,A(y)\,\sin(\phi_h-\phi_R)\,\mathcal{F}\left[\hat{h}\cdot\boldsymbol{k}_T\,\frac{g_1\,G_1^\perp}{M_1M_2}\right] + \ldots\right\} , \tag{40}$$

where ϕ_h is the azimuthal angle of $\boldsymbol{P}_{h\perp}$ (analogously to ϕ_1 in Fig. 1), and λ is the target helicity.

This is a good point to make the connection to handedness studies. Handedness has been studied for quite some time [16, 17, 18] as a means to probe the helicity of fragmenting quarks. Clearly, $G_1^\perp$ is a similar analyzer of this helicity due to a $(\boldsymbol{k}_T\times\boldsymbol{R}_T)$ correlation present in the fragmentation process and a direct link with the concept of longitudinal jet handedness [17] can be made. One can show that the functions appearing in Eq. (11) of Ref. [17] are related to the IFFs dicussed here. For instance, the longitudinal jet handedness is a linear combination of functions called D_1^A and D_2^A and is directly proportional to $(\boldsymbol{k}_T\times\boldsymbol{R}_T)\,G_1^\perp(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$. Similarly, the transversal jet handedness (given by a function called D_1^T) is proportional to $(\boldsymbol{k}_T\times\boldsymbol{R}_T)\,H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$. Although the unintegrated functions $G_1^\perp$ and $H_1^{\sphericalangle}$ are directly related to the jet handedness functions of Ref. [17], the asymmetries we have presented here are not easily translated to the handedness correlation observables discussed in Ref. [11] (different methods of weighting are employed). Nevertheless, they should encode similar information and as such our asymmetry of Eq. (38) could perhaps also serve as a measure of a CP-violating effect of the QCD vacuum discussed in Ref. [11]. This interesting topic deserves further study.

The function $G_1^\perp(z,M_h^2)$ of Eq. (35) also provides a probe of the transverse momentum dependent distribution function g_{1T} through asymmetries in the processes $e\,p^\uparrow\to(h_1h_2)X$ or $p\,p^\uparrow\to(h_1h_2)X$. However, these are precisely the processes where also the transversity asymmetries (proportional to $H_1^{\sphericalangle}$ and $H_1^\perp$) occur. In fact, the cross section at leading twist for the two-hadron semi-inclusive DIS on a transversely polarized target contains the following terms (see Eq. (10) of Ref. [7]):

$$\begin{aligned}
\frac{d\sigma}{d\Omega\,dx\,dz\,d\xi\,d\boldsymbol{P}_{h\perp}\,dM_h^2\,d\phi_R} \propto\; & |\boldsymbol{S}_\perp|\left\{\ldots + |\boldsymbol{R}_T|\,B(y)\,\sin(\phi_R+\phi_{S_\perp})\,\mathcal{F}\left[\frac{h_1\,H_1^{\sphericalangle}}{M_1+M_2}\right]\right.\\
& + B(y)\,\sin(\phi_h+\phi_{S_\perp})\mathcal{F}\left[\hat{h}\cdot\boldsymbol{k}_T\,\frac{h_1\,H_1^\perp}{M_1+M_2}\right] + B(y)\,\cos(\phi_h+\phi_{S_\perp})\mathcal{F}\left[\hat{g}\cdot\boldsymbol{k}_T\,\frac{h_1\,H_1^\perp}{M_1+M_2}\right]\\
& - |\boldsymbol{R}_T|\,A(y)\,\cos(\phi_h-\phi_{S_\perp})\,\sin(\phi_h-\phi_R)\,\mathcal{F}\left[\hat{h}\cdot\boldsymbol{k}_T\,\hat{h}\cdot\boldsymbol{p}_T\,\frac{g_{1T}\,G_1^\perp}{MM_1M_2}\right]\\
& + |\boldsymbol{R}_T|\,A(y)\,\sin(\phi_h-\phi_{S_\perp})\,\sin(\phi_h-\phi_R)\,\mathcal{F}\left[\hat{h}\cdot\boldsymbol{k}_T\,\hat{g}\cdot\boldsymbol{p}_T\,\frac{g_{1T}\,G_1^\perp}{MM_1M_2}\right]\\
& - |\boldsymbol{R}_T|\,A(y)\,\cos(\phi_h-\phi_{S_\perp})\,\cos(\phi_h-\phi_R)\,\mathcal{F}\left[\hat{g}\cdot\boldsymbol{k}_T\,\hat{h}\cdot\boldsymbol{p}_T\,\frac{g_{1T}\,G_1^\perp}{MM_1M_2}\right]\\
& \left. + |\boldsymbol{R}_T|\,A(y)\,\sin(\phi_h-\phi_{S_\perp})\,\cos(\phi_h-\phi_R)\,\mathcal{F}\left[\hat{g}\cdot\boldsymbol{k}_T\,\hat{g}\cdot\boldsymbol{p}_T\,\frac{g_{1T}\,G_1^\perp}{MM_1M_2}\right]\ldots\right\} ,
\end{aligned} \tag{41}$$

where M is the target mass with momentum $P^+ = xp^+$. Hence, one should carefully project out the azimuthal asymmetry of interest in order to avoid contributions from different mechanisms.

VI. CONCLUSIONS

We have studied azimuthal asymmetries in the process $e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X$, which function as probes for interference fragmentation functions (IFFs). The asymmetries arise in the orientation of the two hadron pairs with respect to each other. We have presented two asymmetries that are of present-day experimental relevance. Although the asymmetries probe the correlation of longitudinal and transverse quark and antiquark spin, respectively, they are to be extracted from the same experimental data by applying different weights in the form of trigonometric functions of azimuthal angles. The first asymmetry has already been studied by Artru and Collins, but had not yet been expressed in terms of the IFF language of Refs. [3, 7, 8]. We have also indicated a relation between the function $H_1^{\triangleleft}$, that occurs in this asymmetry, and transversal jet handedness.

The second azimuthal asymmetry that we focussed on specifically, has not been pointed out before and involves the longitudinally polarized quark IFF $G_1^{\perp}$, which is related to longitudinal jet handedness. The asymmetry offers a different way of studying handedness correlations and, as such, can perhaps be used as a measure of a specific CP-violating effect of the QCD vacuum. We pointed out that the knowledge of the helicity distribution function g_1 is of help in this respect.

Extracting IFFs from e^+e^- annihilation will provide for the as yet unknown information needed to disentangle the tranversity distribution from processes like $e\,p^{\uparrow} \to (h_1h_2)X$ or $p\,p^{\uparrow} \to (h_1h_2)X$. However, we stress that azimuthal asymmetries in these processes with transversely polarized targets involve combinations like $g_{1T}G_1^{\perp}$, $h_1H_1^{\triangleleft}$ and $h_1H_1^{\perp}$, hence, a careful separation of each contribution requires weighting of the cross section with the appropriate trigonometric functions.

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